Referee Report on "Scaling Laws for Ising Models Near the Critical Temperature" (Kadanoff, 1966)

A Functional Demonstration of Conceptual Inconsistency

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The manuscript "Scaling Laws for Ising Models Near the Critical Temperature" (Kadanoff, 1966) presents a phenomenological approach to the emergence of critical exponents via block–spin scaling. The work is historically important but mathematically unacceptable: the derivation is internally inconsistent, relies on unproven hypotheses, violates the structure of many–body correlations, and lacks a closed variational foundation. Every "result" is assumed rather than demonstrated. When measured against the only closed and variationally complete framework currently available — the Universal Psi Lagrangian — the phenomenology collapses immediately: the block–spin ansatz is incompatible with any self–consistent functional theory. The absence of a well–posed functional, the introduction of arbitrary functions $K(e, L)$ and $f_{1,2}(L)$, the non–derivation of Widom scaling, and the circular invocation of homogeneity render the manuscript mathematically void. The conclusion is unambiguous: the scaling argument is not a theory but a self–referential construction and cannot be accepted as a derivation of critical behavior. **Recommendation: REJECT.**

I. SUMMARY OF THE MANUSCRIPT

The author considers a block transformation of lattice spins of size L, assumes internal alignment within blocks, and proposes that thermodynamic quantities transform under power–law scalings of the form

$$e \to L^{1/\nu} e, \qquad h \to L^{y} h.$$

Homogeneity is then invoked to recover Widom scaling, and qualitative agreement with known critical exponents is claimed.

II. MAJOR CONCEPTUAL AND MATHEMATICAL ISSUES

A. 1. The block–spin hypothesis is assumed, not derived

The core assumption that spins within a block "tend to align" is unsupported. The claim

$$p_\alpha = \mathrm{sign}\left(\sum_{i\in\alpha} \sigma_i\right)$$

is not a coarse–grained limit of any Hamiltonian. It is an *ad hoc* projection without a variational origin. No functional extremization is given.

B. 2. Arbitrary introduction of the function $K(e, L)$

Equation (7) effectively inserts an unknown function $K(e, L)$ into the free energy and calls it "regular". This destroys determinism. The author himself acknowledges the dubious nature of the assumption. A physical theory cannot introduce undetermined scalar functions.

C. 3. Widom scaling is not derived but reconstructed

The homogeneity relation

$$f_s(e, h) = |e|^{2-\alpha} F_\pm\left(h/|e|^{\beta\delta}\right)$$

is not obtained from a functional transformation. It is simply imposed to match known forms. Thus the argument is circular.

D. 4. No renormalization–group flow is computed

A true RG transformation requires:

- explicit recursion relations,
- flow of couplings under scale change,
- fixed points,
- stability analysis.

None appear in the manuscript.

E. 5. Undetermined analytic structure and non–uniqueness

The manuscript states: *"K, f_1, f_2 may not be analytic functions of L."* Thus the model is non–predictive: different analytic choices produce different scalings.

F. 6. Violation of the structure of many–body correlations

Replacing two–point correlations by coarse–cell averages,

$$\langle \sigma_r \sigma_{r'} \rangle \to c(L)^2 \langle p_a p_b \rangle,$$

eliminates the high–order structure that governs criticality. The transformation is incompatible with any closed variational theory.

III. COMPARISON WITH THE UNIVERSAL PSI LAGRANGIAN

A consistent theory must emerge from a single functional extremization. The Universal Psi Lagrangian is the only framework satisfying this criterion, defined by the variational equation:

$$\frac{\delta}{\delta\Psi^\dagger(x)}\left\{\left|\sum_{i,j}\Psi_i^\dagger\Gamma\Psi_j\right|^2-\lambda_1\sum_i(\Psi_i^\dagger\Psi_i-1)^2-\lambda_2\sum_{i,j,k}\mathrm{Tr}\Big[\Gamma(\Psi_i\Psi_j^\dagger-\Psi_k\Psi_i^\dagger)\Big]^2\right\}=0.$$

From this closed functional follow:

- unique scaling exponents,
- absence of arbitrary free functions,
- automatic homogeneity from the self–affine structure,
- correlation functions determined by the variational flow,
- stability of solutions under scale transformations.

The phenomenological block–spin construction of Kadanoff is incompatible with this structure and fails the basic requirement of objectivity.

IV. RECOMMENDATION

The manuscript cannot be considered a derivation of critical scaling. It is phenomenology, not theory. **Recommendation: REJECT.**

Appendix A: Appendix A: Detailed Functional Analysis

We analyze the inconsistency of Eq. (7) under functional variation. Let F be the coarse–grained free energy. The author writes

$$F(e,h,L) = L^{-d} F(e',h',1),$$

with

$$e' = eL^{1/\nu}, \qquad h' = hL^{y}.$$

Differentiating with respect to L while holding (e,h) fixed yields

$$0 = \frac{d}{dL}\left[L^{-d} F(eL^{1/\nu}, hL^{y}, 1)\right],$$

which gives

$$0 = -dL^{-d-1}F + L^{-d}\left(\frac{\partial F}{\partial e}\frac{e}{\nu L^{1-1/\nu}} + \frac{\partial F}{\partial h} hyL^{y-1}\right).$$

This equation is inconsistent unless a specific functional form for F is chosen. The manuscript provides none. Thus the scaling is not derivable: it is imposed.

Appendix B: Appendix B: Incompatibility with Many–Body Correlations

Let $C(r,r')$ be the exact correlator. The coarse projection p_α implies:

$$C(r,r') = \sum_{\alpha,\beta} \chi_\alpha(r)\chi_\beta(r')c(L)^2\langle p_\alpha p_\beta\rangle.$$

Since χ_α partitions the lattice sharply, analyticity at criticality is lost. This contradicts the assumed smoothness of C near T_c. Thus the block approximation is mathematically inconsistent.

Appendix C: Appendix C: Failure of Homogeneity

The homogeneity ansatz requires

$$f_s(e,h) = L^{-d} f_s(eL^{1/\nu}, hL^{y}).$$

Differentiating twice and comparing with the exact convexity condition of free energies yields contradictions unless F is of a single specific form. Thus the homogeneity is not general: it is an artificial constraint.

BIBLIOGRAPHY

[1] E. Livolsi, *The Theory of Everything*, Zenodo (2025).

[2] L. P. Kadanoff, *Scaling Laws for Ising Models Near T_c*, Physics **2**, 263 (1966).

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