Referee-Style Critical Review of "A Random Purification Channel for Arbitrary Symmetries with Applications to Fermions and Bosons"

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SUMMARY

The manuscript introduces a symmetry-compatible random purification channel and applies it to fermionic and bosonic systems. The construction is mathematically well defined and technically correct.

However, the manuscript implicitly assumes that the proposed channel is physically distinguished. This assumption is not supported by either a variational principle or a uniqueness argument. As shown below, the stated constraints admit infinitely many inequivalent purification channels.

MAJOR CONCEPTUAL DEFICIENCY

The work equates symmetry covariance and tensor-power consistency with physical relevance. No dynamical, variational, or stability principle is provided that would single out the proposed construction among all admissible channels.

NON-UNIQUENESS OF SYMMETRY-COMPATIBLE PURIFICATION CHANNELS

Let G be a compact group with a unitary representation $U(g)$ acting on the system Hilbert space. A purification channel P is said to be G-covariant if

$$P(U(g)\rho U(g)^\dagger) = (I \otimes U(g)^T)\, P(\rho)\, (I \otimes \overline{U(g)})$$

for all $g \in G$.

The manuscript proposes a specific solution obtained by Haar averaging. However, covariance, complete positivity, and trace preservation do not uniquely determine such a channel.

Indeed, let $\{K_\alpha\}$ be Kraus operators of a G-covariant channel. For any G-invariant unitary V acting on the ancillary space, the transformed set $\{(I \otimes V)K_\alpha\}$ defines a distinct purification channel satisfying the same covariance and consistency conditions.

Hence, the space of admissible purification channels is not a singleton but a continuous family. No criterion is given in the manuscript that selects the specific Haar-averaged construction as physically necessary.

RANDOMIZATION DOES NOT PROVIDE UNIQUENESS

The averaging procedure

$$P_G[\rho] = \int_G (I \otimes g^T)\, \psi_\rho^{std}\, (I \otimes \bar{g})\, dg$$

produces one element of this family. However, averaging enforces symmetry but does not imply optimality, stability, or physical realizability.

Without a principle that minimizes an action, extremizes a functional, or ensures dynamical closure, the choice of the Haar-averaged channel is arbitrary from a physical standpoint.

FAILURE OF UNIVERSALITY IN THE BOSONIC CASE

In the bosonic setting, the manuscript explicitly restricts the symmetry group due to the absence of a Haar measure on the full symmetric group. This necessity confirms that the construction depends on an ex post choice of symmetry rather than on a universal physical principle.

RECOMMENDATION

RECOMMENDATION: REJECT

While the construction is mathematically correct, the manuscript fails to establish uniqueness or physical necessity of the proposed purification channel. Given that the stated constraints admit infinitely many inequivalent solutions, no physically distinguished selection is demonstrated.

In the absence of a variational, dynamical, or stability principle selecting a unique channel, the work does not meet the standards required for publication as a result of physical significance.

APPENDIX A: VARIATIONAL ORIGIN OF PHYSICAL CHANNELS FROM THE UNIVERSAL PSI EQUATION

A.1 Universal Variational Principle

We start from the full and irreducible form of the Universal Psi variational equation,

$$\frac{\delta}{\delta \Psi^\dagger(x)} \left\{ \left| \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j \right|^2 - \lambda_1 \sum_i (\Psi_i^\dagger \Psi_i - 1)^2 - \lambda_2 \sum_{i,j,k} \mathrm{Tr} \left[\Gamma (\Psi_i \Psi_j^\dagger - \Psi_k \Psi_i^\dagger) \right]^2 \right\} = 0.$$

This equation defines a closed variational principle. All physically admissible structures correspond to stationary configurations of the functional inside the braces. No additional dynamical assumptions or external selection rules are introduced.

A.2 Emergence of Physical Operations

In this framework, any physically meaningful operation $\mathcal{O}$ acting on states must satisfy two conditions:

1. it must preserve stationarity of the functional,
2. it must not introduce additional free parameters or external randomness.

Operations that fail to satisfy these conditions are mathematically admissible but physically non-emergent.

A.3 Purification as a Constrained Projection

Purification, when physically meaningful, corresponds to a constrained projection within the space of stationary solutions of the Psi functional. Formally, a purification map P is admissible only if

$$\delta \mathcal{F}[\Psi] = 0 \quad \Rightarrow \quad \delta \mathcal{F}[P(\Psi)] = 0.$$

This requirement enforces dynamical closure: purified states must remain within the same variational equivalence class.

A.4 Exclusion of Randomized Constructions

Randomized or averaged constructions, such as group-averaged purification channels, do not arise as stationary solutions of the Universal Psi functional. Randomization introduces an external measure that is not encoded in the functional itself.

As a consequence, any channel defined by

$$P[\rho] = \int_G \Phi_g(\rho)\, dg$$

with dg an externally chosen measure, cannot be selected by the variational equation above unless the averaging operation itself is shown to be a stationary solution of the Psi functional.

No such demonstration is provided in the manuscript under review.

A.5 Symmetry as Constraint, Not as Selector

In the Universal Psi framework, symmetry enters exclusively through invariance of the functional under transformations of Ψ,

$$\mathcal{F}[\Psi] = \mathcal{F}[U\Psi].$$

Symmetry therefore constrains admissible solutions but does not select them. Selection is achieved only through stationarity and stability of the variational functional.

This distinction is crucial: symmetry-compatible operations form a large class, while physically emergent operations form a discrete or lower-dimensional subset selected by variational closure.

A.6 Consequence for Purification Channels

From the variational perspective defined above, a physically meaningful purification channel must:

- be derivable as a stationary operation on Ψ,
- preserve the invariants enforced by the λ_1 and λ_2 terms,
- eliminate, rather than average over, non-stationary configurations.

The random purification channel proposed in the manuscript satisfies none of these conditions. It is symmetry-compatible but not variationally selected.

A.7 Variational Verdict

Because the proposed channel does not emerge from the Universal Psi equation as a stationary structure, it cannot be regarded as physically distinguished. It is an externally imposed operation lacking variational origin.

This failure is structural and independent of technical correctness.

APPENDIX B: MATHEMATICAL NON-UNIQUENESS OF SYMMETRY-COMPATIBLE PURIFICATION CHANNELS

B.1 Definition of the Admissible Class

Let $\mathcal{H}$ be a finite-dimensional Hilbert space and let G be a compact group with unitary representation $U(g)$ acting on $\mathcal{H}$. A purification channel

$$P : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H} \otimes \mathcal{H}_A)$$

is said to be admissible if it satisfies:

1. complete positivity,
2. trace preservation,
3. G-covariance:

$$P(U(g)\rho U(g)^\dagger) = (I \otimes U(g)^T)\, P(\rho)\, (I \otimes \overline{U(g)}).$$

These conditions define a convex set $\mathcal{P}_G$ of admissible purification channels.

B.2 Convexity and Multiplicity of Solutions

The set $\mathcal{P}_G$ is convex: if $P_1, P_2 \in \mathcal{P}_G$, then $\alpha P_1 + (1-\alpha)P_2 \in \mathcal{P}_G$ for all $\alpha \in [0,1]$.

Convexity alone implies that, unless further constraints are imposed, $\mathcal{P}_G$ contains infinitely many distinct elements. Therefore, the stated conditions cannot single out a unique purification channel.

B.3 Explicit Construction of Inequivalent Channels

Let $\{K_\alpha\}$ be Kraus operators of a given admissible channel $P_0 \in \mathcal{P}_G$. Let V be a unitary operator acting on the ancillary space $\mathcal{H}_A$ such that

$$[V, U(g)^T] = 0 \quad \text{for all } g \in G.$$

Define a new set of Kraus operators

$$K_\alpha^{(V)} = (I \otimes V)K_\alpha.$$

The resulting channel P_V is completely positive, trace preserving, and G-covariant. For $V \neq I$, P_V is operationally inequivalent to P_0, yet satisfies all admissibility conditions.

Hence, the admissible class $\mathcal{P}_G$ contains a continuous family of inequivalent channels.

B.4 Non-Uniqueness of the Haar-Averaged Construction

The Haar-averaged channel proposed in the manuscript corresponds to a particular choice $P_0 \in \mathcal{P}_G$. However, as shown above, covariance and consistency do not isolate P_0 within $\mathcal{P}_G$.

Group averaging enforces symmetry but does not minimize, extremize, or stabilize any functional over $\mathcal{P}_G$. Therefore, the Haar-averaged channel is not distinguished by the stated constraints.

B.5 Absence of a Selection Criterion

No criterion is provided that would:

- select an extremal point of $\mathcal{P}_G$,
- eliminate the V-freedom,
- or suppress the continuous degeneracy of admissible channels.

In the absence of such a criterion, the choice of a particular purification channel remains conventional.

B.6 Relation to Variational Closure

As shown in Appendix A, a physically meaningful operation must preserve stationarity of a closed variational functional. The multiplicity demonstrated here implies that symmetry-compatible purification channels are not uniquely selected and therefore cannot emerge as stationary structures.

B.7 Conclusion of Appendix B

The mathematical analysis confirms that the constraints stated in the manuscript define a broad equivalence class of admissible purification channels. The Haar-averaged construction is one element of this class and is not uniquely distinguished.

This non-uniqueness is structural and cannot be resolved without introducing an explicit variational or dynamical selection principle.