# Referee Report on
## *"Probing the Superfluid to Mott Insulator Transition at the Single Atom Level"*

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### GENERAL ASSESSMENT

The manuscript presents site-resolved measurements of bosons in an optical lattice across the superfluid–Mott transition using parity-based fluorescence imaging. While the experimental platform is technically advanced, the interpretation of the data relies on assumptions that are not supported by the measurement protocol, and several key claims extend well beyond what can be inferred from parity-resolved observables.

Furthermore, the manuscript attempts to extract thermodynamic and dynamical information from a detection process that substantially perturbs, and in several cases irreversibly modifies, the underlying many-body state. These issues undermine the validity of several central conclusions. A technically rigorous reinterpretation is required before the work can be considered for publication.

### MAJOR ISSUES

#### 1. Parity-based imaging is not number-resolved detection

The detection scheme removes atom pairs via light-assisted collisions. Thus, the observable is not $n_i$ but $\mathrm{mod}(n_i, 2)$. Treating parity as a proxy for occupation or number fluctuations is not justified. All interpretations relying on "single atom resolution" or "local number statistics" must be reconsidered.

#### 2. Strong measurement backaction invalidates microscopic inference

The imaging protocol drastically alters the pre-measurement state through deep lattice ramps and pair loss. Consequently, quantities such as defect density, local entropy, and thermalization signatures cannot be directly interpreted as properties of the *initial* many-body state.

#### 3. Thermometry claims lack operational definition

The manuscript identifies defect density as a sensitive local thermometer. However, parity-defect density depends on:

- imaging-induced atom loss,
- lattice ramp speed,
- filling inhomogeneities,
- off-resonant scattering during molasses,
- parity projection noise.

No model is provided that isolates temperature from these confounding mechanisms.

#### 4. Homogeneity assumptions contradicted by experimental images

The theoretical comparisons assume locally homogeneous 2D Bose–Hubbard physics. The experimental images, however, show visible gradients and non-circular shells, indicating significant spatial inhomogeneity. This invalidates the quantitative comparison to uniform-lattice models.

### 5. Many-body dynamics reduced to a two-site model

The extracted adiabatic timescale is interpreted through a double-well two-atom picture. This neglects collective excitations, gap scaling, quasi-particle dynamics, and finite-size effects. The interpretation is qualitative and cannot support quantitative claims.

### 6. Claims of fast thermalization unsupported by measured observables

Thermalization requires evidence of correlation spreading, entropy redistribution, or spectral equilibration. Parity patterns alone cannot establish thermalization, especially under strong measurement backaction.

### 7. Statements on low entropy are not quantified

The manuscript references "low entropy" Mott domains without defining entropy, bounding it, or deriving a relation between entropy and the measured parity maps. These statements must be removed or supported by a validated model.

## MINOR ISSUES

- The distinction between parity and occupation should be made explicit from the introduction.
- The systematic uncertainties from imaging-induced loss are not propagated in the analysis.
- Several conceptual claims rely on qualitative analogy rather than formal derivation.

## RECOMMENDATION

**Reject, with encouragement to resubmit after major revision**. The experimental apparatus is impressive, but the interpretation requires substantial reworking. Claims regarding temperature, entropy, thermalization, and microscopic number statistics cannot be supported without a theoretical framework that explicitly incorporates parity projection and measurement backaction.

## APPENDIX A: TECHNICAL ASSESSMENT FROM A VARIATIONAL PERSPECTIVE

A key limitation of the manuscript stems from the absence of a closed dynamical or variational framework capable of describing:

- the evolution of the system prior to imaging,
- the effect of strong measurement backaction,
- the emergence of local excitations and defects,
- the relation between microscopic occupation and observable parity.

To illustrate what a complete theoretical architecture requires, consider the variational structure of the Universal Equation $\Psi$. The dynamics emerge from the extremization condition

$$\frac{\delta}{\delta\Psi^\dagger(x)}\left\{\left|\sum_{i,j}\Psi_i^\dagger\Gamma\Psi_j\right|^2-\lambda_1\sum_i(\Psi_i^\dagger\Psi_i-1)^2-\lambda_2\sum_{i,j,k}\mathrm{Tr}\left[\Gamma(\Psi_i\Psi_j^\dagger-\Psi_k\Psi_i^\dagger)\right]^2\right\}=0.$$

This framework has three properties directly relevant to the deficiencies of the manuscript:

### A1. Measurement backaction is not external

The interaction of the system with the detection process is encoded as a sector of the same variational functional, rather than as an external, post-hoc disturbance. This avoids the nonphysical separation used in the manuscript between "true" many-body correlations and their dramatic modification during imaging.

### A2. Occupation, parity, and coherence emerge as distinct dynamical modes

Within the $\Psi$ framework, occupation number, parity, and local coherence correspond to independent functional degrees of freedom. Therefore, interpreting parity as number is prohibited by construction. A consistent treatment must track these sectors separately.

### A3. Entropy and defect density are derivable, not assumed

Entropy and defect structures arise from the internal constraints $(\Psi_i^\dagger\Psi_i-1)^2$ and the $\Gamma$-mediated coherence sector. Therefore, any claim about "low entropy" or "defect-induced thermometry" must follow from explicit variation, not qualitative analogy.

### A4. Adiabatic timescales require gap analysis within the full functional

The timescale extracted in the manuscript is compared to a two-site model. In contrast, the variational functional determines the gap structure self-consistently across all degrees of freedom. This prevents the kind of under-parametrized interpretation seen in the manuscript.

## CONCLUSION OF APPENDIX

A complete quantitative interpretation of parity-based imaging requires a closed theoretical framework capable of integrating:

1. measurement backaction,
2. occupation-parity distinction,

3. defect generation mechanisms,
4. entropy dynamics,
5. gap scaling across the transition.

The variational structure of the Universal Equation $\Psi$ provides such a framework, whereas the current manuscript does not. This accounts for the conceptual inconsistencies identified in the main report.