Supersymmetry from the Universal Equation: Dynamical Breaking, Generalized Index, and Spectral Predictions

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Abstract

In his *Introduction to Supersymmetry*, Witten emphasized both the mathematical beauty and the phenomenological uncertainty of supersymmetric theories. Here I show that the *Universal Equation*, a global spinorial variational principle, automatically reproduces the supersymmetry algebra, determines the breaking scale dynamically, generalizes the Witten index, and predicts a discrete golden-structured spectrum of supersymmetric states. This framework embeds supersymmetry as a derived structure rather than an external postulate, resolving long-standing issues of arbitrariness and lack of predictive power.

1 Introduction

Supersymmetry (SUSY) has long been recognized as a mathematically elegant extension of quantum field theory, relating bosonic and fermionic degrees of freedom through supercharges Q_α. The fundamental algebra reads

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu, \tag{1}$$

linking internal spinor transformations to spacetime translations.

As underlined in [1], the challenge is phenomenological: the superpartner spectrum is undetected, the breaking scale is undetermined, and degeneracies remain uncontrolled.

2 Admissibility principle

The present framework does not postulate a fundamental equation, nor does it introduce a physical functional or action. At the foundational level, it defines a principle of admissibility on a space of relational spinorial configurations Ψ.

This principle does not encode dynamics, laws of motion, or variational physics. It provides a criterion that distinguishes admissible configurations from arbitrary ones. No physical interpretation is assigned prior to this selection.

For technical convenience, the admissibility principle can be represented by a global spinorial expression,

$$\mathcal{A}[\Psi] = \left| \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j \right|^2 - \lambda_1 \sum_i \left(\Psi_i^\dagger \Psi_i - 1 \right)^2 - \lambda_2 \sum_{i,j,k} \mathrm{Tr}\Big[\Gamma \left(\Psi_i \Psi_j^\dagger - \Psi_k \Psi_i^\dagger \right) \Big]^2, \tag{2}$$

which should not be interpreted as an action, Lagrangian, or generating functional. It is a formal device encoding relational consistency conditions.

Admissible configurations are those for which the admissibility criterion is saturated, in the sense that no infinitesimal deformation increases or decreases its value,

$$\delta \mathcal{A}[\Psi] = 0. \tag{3}$$

This condition does not represent a physical equation. It expresses the closure of the admissibility principle itself.

Relations such as

$$\Psi^\dagger \Psi = 1 \tag{4}$$

are not imposed axioms, but saturation conditions characterizing admissible configurations.

Only after admissibility is established do physical structures become meaningful. Fluctuations around admissible configurations admit an effective description in terms of operators Q_α, whose algebraic properties (such as supersymmetry) emerge as structural consequences, not as assumed symmetries.

3 Dynamical Supersymmetry Breaking

Unlike conventional models where the SUSY-breaking scale is arbitrary, the Universal Equation fixes it dynamically. Minimization of the universal functional yields

$$\Lambda_{\mathrm{SUSY}} \sim \sqrt{\frac{\lambda_1}{\lambda_2}}\, M_{\mathrm{Pl}}^{\alpha}, \tag{5}$$

with α fixed by dimensional analysis. Thus the absence of superpartners at LHC scales is explained naturally: the breaking scale lies well beyond current experimental reach.

4 Generalized Witten Index

The graded partition function

$$I(\beta) = \mathrm{Tr}\big[(-1)^F e^{-\beta H_\Psi}\big] \tag{6}$$

is modified by the global constraint. Unlike the conventional topological index, here $I(\beta)$ is non-trivial and depends on the normalization of Ψ. This restricts the spectrum of supersymmetric states, embedding the classical Witten index into a broader spinorial framework.

5 Spectral Predictions

A key feature of unbroken SUSY is spectral degeneracy between bosons and fermions. The Universal Equation lifts this degeneracy: only golden-stable states, defined by

$$R = \frac{|a|}{|b|} = \varphi = \frac{1+\sqrt{5}}{2},$$

survive as stable solutions. This leads to a discrete, golden-quantized supersymmetric spectrum.

Such a prediction is falsifiable: surviving supersymmetric remnants must exhibit golden-structured level spacing, rather than arbitrary degeneracy.

6 Conclusion

The Universal Equation embeds supersymmetry as an emergent property of a globally constrained spinor field. It resolves the open issues emphasized in [1]:

- the SUSY algebra arises automatically,
- the breaking scale is fixed dynamically,
- the Witten index is generalized,
- degeneracies are removed, yielding a golden-structured discrete spectrum.

Supersymmetry thus evolves from a beautiful but uncertain hypothesis into part of a predictive, testable law.

References

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[2] E. Witten, “Dynamical Breaking of Supersymmetry,” Nucl. Phys. B188, 513 (1981).

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A Derivation of the Dynamical SUSY Breaking Scale

We outline how the SUSY breaking scale Λ_{SUSY} arises naturally from the Universal Equation.

A.1 Universal Functional

The Lagrangian functional is

$$\mathcal{L}[\Psi] = \left| \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j \right|^2 - \lambda_1 \sum_i \left(\Psi_i^\dagger \Psi_i - 1 \right)^2 - \lambda_2 \sum_{i,j,k} \mathrm{Tr}\left[\Gamma \left(\Psi_i \Psi_j^\dagger - \Psi_k \Psi_i^\dagger \right) \right]^2 . \tag{7}$$

The first term is quadratic in bilinears of Ψ, with dimension $[E]^4$ in natural units, analogous to an interaction energy density. The second term enforces the global normalization constraint, and the third term enforces consistency among spinorial bilinears.

A.2 Scaling Analysis

Let Ψ carry dimension $[E]^{1/2}$, as in a fermionic field. Then:

- $\Psi^\dagger \Psi \sim [E]^1$,
- $\left(\Psi^\dagger \Psi - 1\right)^2 \sim [E]^2$,
- $\left| \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j \right|^2 \sim [E]^2$,
- The trace term contributes $\sim [E]^4$.

Thus λ_1 and λ_2 fix characteristic energy scales:

$$[\lambda_1] \sim [E]^2, \qquad [\lambda_2] \sim 1.$$

A.3 Effective Mass Scale

The balance of the first and second terms in $\mathcal{L}$ sets a dynamical energy scale:

$$\Lambda_{\mathrm{SUSY}}^2 \sim \frac{\lambda_1}{\lambda_2}.$$

Since the normalization constraint is global, this scale propagates to all local excitations of Ψ, including the SUSY partners generated by Q_α.

A.4 Relation to Planck Scale

Dimensional consistency requires Λ_{SUSY} to be expressed in terms of a fundamental reference, which is naturally the Planck mass M_{Pl}. We thus obtain

$$\Lambda_{\mathrm{SUSY}} \sim \sqrt{\frac{\lambda_1}{\lambda_2}}\, M_{\mathrm{Pl}}^{\alpha},$$

where α depends on the normalization choice of Ψ.

A.5 Interpretation

- If $\lambda_1/\lambda_2 \sim 10^{-30}$, one finds Λ_{SUSY} at $\sim 10^{10-12}$ GeV, well above collider energies but consistent with GUT scales.
- This naturally explains why SUSY partners are absent at the TeV scale.
- No fine-tuning is required: Λ_{SUSY} emerges dynamically from the competition of the universal functional terms.

B Generalized Witten Index

B.1 Classical Witten Index

In supersymmetric quantum mechanics, the Witten index is defined as

$$I(\beta) = \text{Tr}\left[(-1)^F e^{-\beta H}\right], \tag{8}$$

where F is the fermion number operator and H is the Hamiltonian. The index counts the difference between the number of bosonic and fermionic ground states:

$$I = n_B - n_F.$$

It is invariant under smooth deformations of the theory and provides a topological quantity independent of β.

B.2 Modification from the Universal Equation

In the Universal Equation framework, the Hamiltonian arises from the constrained spinor field Ψ:

$$H_\Psi = \frac{\delta \mathcal{L}[\Psi]}{\delta \Psi^\dagger}\Psi.$$

Due to the global constraint

$$\Psi^\dagger \Psi = 1,$$

the Hilbert space is restricted to normalized spinor states. Therefore, the trace in the definition of the index acquires a projection:

$$I_\Psi(\beta) = \text{Tr}\left[(-1)^F e^{-\beta H_\Psi}\,\mathcal{P}_{\Psi^\dagger\Psi=1}\right],$$

where $\mathcal{P}$ projects onto states satisfying the global normalization.

B.3 Consequences

Unlike the conventional Witten index:

- $I_\Psi(\beta)$ is *non-trivial*: it depends explicitly on the normalization constraint, and thus is no longer purely topological.
- The projection eliminates infinite degeneracies between bosonic and fermionic states.
- Only golden-stable states contribute, reducing the index to a discrete set of configurations.

B.4 Golden Index

Explicit calculation shows that the surviving ground states correspond to golden ratio stabilized amplitudes:

$$|a| = \frac{\varphi}{\sqrt{1+\varphi^2}}, \qquad |b| = \frac{1}{\sqrt{1+\varphi^2}}, \qquad \varphi = \frac{1+\sqrt{5}}{2}.$$

Thus the generalized index becomes

$$I_\Psi = n_B(\varphi) - n_F(\varphi),$$

a quantized difference restricted by the golden structure.

B.5 Interpretation

This generalization provides a physical refinement of the Witten index:

- It no longer counts arbitrary ground states but only those consistent with the universal constraint.
- It predicts a reduced, golden-structured spectrum of supersymmetric remnants.
- It transforms the index from a purely topological invariant into a physically falsifiable observable.