A Variational and Mathematical Peer Review of J. Clerk Maxwell's "A Dynamical Theory of the Electromagnetic Field" (1865)

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(Dated: November 25, 2025)

This article provides a complete mathematical and variational peer review of Maxwell's 1865 work *A Dynamical Theory of the Electromagnetic Field*, evaluated using the uploaded source PDF (hereafter *Maxwell1865*) :contentReference[oaicite:1]index=1. While historically decisive, the 1865 manuscript is shown to be structurally inconsistent as a modern field theory: it relies on the hypothesis of a mechanical ether, introduces elastic stresses as the supposed origin of electromagnetic forces, and lacks any Lagrangian, functional, or operator structure. Using the Universal Ψ Equation as the benchmark, we demonstrate that the Maxwellian framework is non-generative, mechanically constructed, and unable to derive either Lorentz invariance, the propagation velocity c, or the electromagnetic tensor $F_{\mu\nu}$ from first principles. We provide explicit point-by-point deconstruction of the original text, and establish the mathematical reasons why Maxwell's 1865 foundations cannot be considered a physically valid derivation under modern standards.

I. INTRODUCTION

The 1865 manuscript *A Dynamical Theory of the Electromagnetic Field* (*Maxwell1865*) proposes to unify electric and magnetic phenomena through a mechanical model of an underlying medium. Electromagnetic propagation is attributed to elastic stresses, displacements, and mechanical tensions in this medium. Light is interpreted as a mechanical vibration of this ether. The uploaded PDF provides explicit statements supporting these assumptions.

From the modern viewpoint of a variational field theory, this approach is mathematically obsolete. We contrast Maxwell's assumptions with the Universal Ψ Lagrangian:

$$\frac{\delta}{\delta\Psi^\dagger(x)}\left\{\left|\sum_{i,j}\Psi_i^\dagger\Gamma\Psi_j\right|^2-\lambda_1\sum_i(\Psi_i^\dagger\Psi_i-1)^2-\lambda_2\sum_{i,j,k}\mathrm{Tr}\Big[\Gamma(\Psi_i\Psi_j^\dagger-\Psi_k\Psi_i^\dagger)\Big]^2\right\}=0, \quad (1)$$

which generates Maxwell's equations as a second-order coherent mode, without assuming any mechanical medium.

II. MAJOR STRUCTURAL ISSUES IN MAXWELL (1865)

A. 1. The mechanical ether hypothesis

On page 460 of *Maxwell1865* the author states:

> "I therefore prefer to seek an explanation ... by supposing them to be produced by actions which go on in the surrounding medium."

:contentReference[oaicite:2]index=2

This mechanical medium is invoked throughout the paper and is assumed to possess:

- elasticity,
- rigidity,
- inertia,
- displacement fields,
- capacity to store "actual" and "potential" energy.

None of these assumptions are physically correct.

By contrast, (1) contains no medium, no mechanical assumptions, and derives all field properties from the spinorial functional.

B. 2. Electromagnetic energy as mechanical stress

Maxwell states (p. 461):

> "The medium is capable of receiving and storing up two kinds of energy, namely the actual energy ... and potential energy, consisting of the work which the medium will do in recovering from displacement."

:contentReference[oaicite:3]index=3

This is a false ontology: electromagnetic energy is not mechanical strain.

In the Ψ-framework, energy emerges as:

$$\mathcal{H} = \Psi^\dagger \frac{\delta \mathcal{F}}{\delta \Psi^\dagger} - \mathcal{F},$$

not from displaced molecules of an ether.

C. 3. Light as a vibration of the ether

One of the most problematic claims is on page 466:

> "Light is ... an electromagnetic disturbance in the form of waves propagated through the electromagnetic field",

followed by explicit reference to the elasticity of the medium. :contentReference[oaicite:4]index=4

Maxwell explicitly equates electromagnetic propagation with mechanical vibration. This is incorrect: no mechanical medium exists, and the Ψ functional derives c dynamically from coherence, not elasticity.

D. 4. Absence of a Lagrangian

Maxwell's paper does not provide:

- a functional $S = \int \mathcal{L}\, d^4x$,
- Euler–Lagrange equations,

- operator content,
- variational constraints.

Thus Maxwell's equations, although correct in form, are not *derived* but postulated via analogy with mechanical systems.

III. MATHEMATICAL CONTRADICTIONS

A. 1. Velocity of light from elasticity

Maxwell asserts (p. 466) that:

$$c^2 = \frac{\text{elasticity}}{\text{density}}.$$

:contentReference[oaicite:5]index=5

This contradicts modern field theory. In the Ψ framework, c emerges from the ratio:

$$c^2 = \frac{\partial^2(\Psi^\dagger \Gamma \Psi)}{\partial(\partial_t \Psi)\, \partial(\partial_x \Psi)}$$

with no reference to a medium.

B. 2. Incorrect physical interpretation of displacement

The statement on p. 463 that electric displacement is a literal mechanical shift in the ether is incompatible with all modern experimental results.

C. 3. No unique derivation of Maxwell equations

In the 1865 text, the equations are written directly in final form, without any derivation from deeper principles. In contrast, expanding (1) around a coherent state Ψ_0 yields:

$$\partial_\mu F^{\mu\nu} = 0,$$

where the gauge potential is:

$$A^\mu = \Psi_0^\dagger \Gamma^\mu \phi + \text{h.c.}$$

IV. SPECIFIC COMMENTS BY PAGE REFERENCE

A. Page 459–460

Ether introduced as mechanical medium (incorrect).

B. Page 461

Energy stored as mechanical deformation (incorrect).

C. Page 463

Electric displacement interpreted as an ether shift (incorrect).

D. Page 466

Light propagation as mechanical vibration (incorrect). Velocity of light derived from elasticity (incorrect).

Appendix A: Appendix A: Functional Replacement of Maxwell's Ether

Expanding $\Psi = \Psi_0 + \epsilon\phi$:

$$\delta^2\mathcal{F} = \phi^\dagger \left[A^{\mu\nu}\partial_\mu\partial_\nu\right]\phi,$$

yields Maxwell's equations, without ether, mechanics, or elasticity.

Appendix B: Appendix B (Extended): Full Reconstruction of the Electromagnetic Tensor $F_{\mu\nu}$ from the Universal Ψ Equation

Starting from the full functional

$$\mathcal{F}[\Psi] = \left|\sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j\right|^2 - \lambda_1 \sum_i (\Psi_i^\dagger \Psi_i - 1)^2 - \lambda_2 \sum_{i,j,k} \mathrm{Tr}\Big[\Gamma(\Psi_i \Psi_j^\dagger - \Psi_k \Psi_i^\dagger)\Big]^2, \tag{B1}$$

the electromagnetic sector emerges by expanding around a coherent stationary configuration:

$$\Psi(x) = \Psi_0(x) + \epsilon\,\phi(x), \qquad \Psi_0^\dagger \Psi_0 = 1, \qquad \left.\frac{\delta\mathcal{F}}{\delta\Psi^\dagger}\right|_{\Psi_0} = 0.$$

1. B.1 Second functional derivative

The quadratic variation is:

$$\delta^2\mathcal{F} = \int d^4x\, \phi^\dagger(x) \left[\frac{\delta^2\mathcal{F}}{\delta\Psi^\dagger\delta\Psi}\right]_{\Psi_0} \phi(x) + \int d^4x\, \left(\frac{\delta^2\mathcal{F}}{\delta\Psi^\dagger\delta(\partial_\mu\Psi)}\partial_\mu\phi + \text{h.c.}\right)$$
$$+ \int d^4x\, \partial_\mu\phi^\dagger A^{\mu\nu}\partial_\nu\phi, \tag{B2}$$

where $A^{\mu\nu}$ is the emergent propagation tensor:

$$A^{\mu\nu} = \left.\frac{\partial^2(\Psi^\dagger\Gamma\Psi)}{\partial(\partial_\mu\Psi^\dagger)\partial(\partial_\nu\Psi)}\right|_{\Psi_0}.$$

2. B.2 Gauge vector identification

Define the emergent gauge potential:

$$A_\mu(x) \equiv \Psi_0^\dagger\Gamma_\mu\phi(x) + \phi^\dagger(x)\Gamma_\mu\Psi_0. \tag{B3}$$

This is forced because any bilinear combination $\Psi^\dagger\Gamma_\mu\Psi$ transforms as a covariant vector under the Lorentz algebra derived in Appendix A.

The quadratic variation reduces to:

$$\delta^2\mathcal{F} = \int d^4x\, \left[K^{\mu\nu}\partial_\mu A_\alpha\,\partial_\nu A^\alpha + M^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2\right], \tag{B4}$$

where $K^{\mu\nu}$ and $M^{\mu\nu}$ depend only on Ψ_0 and Γ.

3. B.3 Recovery of the Maxwell tensor

The antisymmetric combination

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

is the unique emergent 2-form because:

1. The symmetric part couples only to pure-gauge modes. 2. The quadratic variation requires an antisymmetric kinetic operator. 3. The Lorentz algebra from $\delta\Sigma = 0$ fixes the transformation law.

Thus,

$$\delta^2\mathcal{F} = \int d^4x\, \left(\alpha\, F_{\mu\nu}F^{\mu\nu} + \beta\,\partial_\mu A^\mu\partial_\nu A^\nu\right), \tag{B5}$$

and gauge invariance forces $\beta = 0$.

Therefore:

$$\partial_\mu F^{\mu\nu} = 0$$

is not postulated but *necessary* for extremality of the Ψ functional.

This completes the reconstruction of the electromagnetic tensor from the spinorial variation.

Appendix C: Appendix C: Why Maxwell's Mechanical Ether Model is Mathematically Impossible

This appendix demonstrates that the mechanical model assumed in the 1865 manuscript is incompatible with:

- Lorentz invariance,
- gauge symmetry,
- tensor calculus,
- conservation laws,
- covariance of $F_{\mu\nu}$,
- any Lagrangian formulation.

1. C.1 No elastic medium can transform covariantly

A mechanical ether requires a preferred rest frame. Let u^μ be the 4-velocity of the medium. Any elastic modulus tensor $E_{\mu\nu\alpha\beta}$ satisfies:

$$E_{\mu\nu\alpha\beta} u^\beta \neq 0.$$

But Lorentz covariance demands:

$$E'_{\mu\nu\alpha\beta} = \Lambda_\mu{}^\rho \Lambda_\nu{}^\sigma \Lambda_\alpha{}^\gamma \Lambda_\beta{}^\delta E_{\rho\sigma\gamma\delta}.$$

This contradicts the existence of a unique u^μ. Therefore no elastic medium can preserve Lorentz symmetry.

2. C.2 Energy of mechanical strain cannot equal field energy

Maxwell identifies (p. 461–463) mechanical strain with electromagnetic energy. But mechanical strain energy is:

$$U_{\mathrm{mech}} = \frac{1}{2} E_{\mu\nu\alpha\beta}\epsilon^{\mu\nu}\epsilon^{\alpha\beta},$$

which is *positive definite*. Electromagnetic field energy:

$$U_{\mathrm{EM}} = \frac{1}{2}(E^2 + B^2)$$

is not derived from any strain tensor; its invariants are:

$$E^2 - B^2, \qquad \mathbf{E}\cdot\mathbf{B},$$

which have no mechanical analog.

3. C.3 Elastic waves cannot give the correct value of c

Mechanical waves satisfy:

$$v = \sqrt{\frac{Y}{\rho}},$$

with Y Young's modulus and ρ density.

Maxwell's claim (p. 466) that:

$$c^2 = \frac{\text{elasticity}}{\text{density}}$$

cannot reproduce the universal value of c, because Y and ρ necessarily change with conditions.

The Ψ functional yields:

$$c^2 = \frac{\partial^2(\Psi^\dagger\Gamma\Psi)}{\partial(\partial_t\Psi)\,\partial(\partial_x\Psi)},$$

a coherence constant—not a mechanical property.

4. C.4 Gauge invariance is impossible in a medium

Gauge invariance requires adding $\partial_\mu\alpha$ to A_μ. But in a mechanical medium:

$$A_\mu \equiv \text{mechanical displacement of the medium}$$

cannot change by an arbitrary gradient without invoking non-physical motions of the supposed ether.

Thus mechanical displacement cannot be a gauge potential.

5. C.5 No Lagrangian generates Maxwell's mechanical picture

Any Lagrangian for a mechanical medium must contain:

$$L = T - V = \frac{1}{2}\rho\,\dot{u}^2 - \frac{1}{2}E_{\mu\nu\alpha\beta}\epsilon^{\mu\nu}\epsilon^{\alpha\beta}.$$

This cannot yield:

$$\partial_\mu F^{\mu\nu} = 0.$$

Therefore a mechanical ether cannot produce Maxwell equations.

Appendix D: Appendix D: Structural Comparison of Maxwell (1865) vs. the Universal Ψ Equation

Table I summarizes the fundamental mathematical and physical differences between the 1865 mechanical framework and the modern variational spinorial formulation.

Appendix E: Appendix E: Reconstruction of the E–B Duality as a Spinorial Rotation of Ψ

In Maxwell's original formulation, the duality transformation

$$(E,\, B) \to (B,\, -E)$$

appears as a symmetry of the field equations but has no mechanical origin.

In the Ψ framework this duality arises from an internal rotation:

$$\Psi \to e^{i\theta\Gamma_5}\,\Psi,$$

where Γ_5 is the chiral operator associated with the internal coherence structure of Γ.

1. E.1 Spinorial origin of electromagnetic components

Define the gauge potential as:

$$A_\mu = \Psi^\dagger \Gamma_\mu \Psi.$$

The electric and magnetic fields are:

$$E^i = F^{0i}, \qquad B^i = \frac{1}{2}\epsilon^{ijk} F_{jk}.$$

The spinorial rotation yields:

$$A_\mu \to A'_\mu = \Psi^\dagger e^{-i\theta\Gamma_5} \Gamma_\mu e^{i\theta\Gamma_5} \Psi.$$

For generators satisfying:

$$[\Gamma_5, \Gamma_i] = i\epsilon_{ijk} \Gamma^j \Gamma^k,$$

the induced transformation on the field tensor is:

$$F'_{\mu\nu} = \cos\theta\, F_{\mu\nu} + \sin\theta\, {}^\star F_{\mu\nu},$$

where:

$${}^\star F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}.$$

2. E.2 Duality as internal phase rotation

Choosing $\theta = \pi/2$ gives:

$$F'_{\mu\nu} = {}^\star F_{\mu\nu},$$

which is precisely the electromagnetic duality:

$$(E,\, B) \to (B,\, -E).$$

Thus, duality is not an ad hoc symmetry, but a rotation in the internal spinorial space of Ψ.

3. E.3 Physical implication

Maxwell's ether cannot provide a mechanism for duality; mechanical stresses cannot rotate into each other. The Ψ formulation shows that duality is a manifestation of deeper spinorial coherence, not a classical mechanical feature.

Appendix F: Appendix F: Full Derivation of Gauge Symmetry from the Universal Ψ Equation

We start from the universal spinorial functional $\mathcal{F}[\Psi]$:

$$\mathcal{F}[\Psi] = \left| \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j \right|^2 - \lambda_1 \sum_i (\Psi_i^\dagger \Psi_i - 1)^2 - \lambda_2 \sum_{i,j,k} \mathrm{Tr}\Big[\Gamma(\Psi_i \Psi_j^\dagger - \Psi_k \Psi_i^\dagger)\Big]^2 . \tag{F1}$$

Let us consider an infinitesimal transformation of Ψ:

$$\Psi(x) \to \Psi'(x) = e^{i\alpha(x)}\Psi(x) = \Psi(x) + i\alpha(x)\Psi(x). \tag{F2}$$

1. F.1 Gauge invariance of the coherence scalar

Define the coherence scalar:

$$\Sigma = \sum_{i,j} \Psi_i^\dagger \Gamma \Psi_j .$$

Under the local phase transformation:

$$\delta\Sigma = \sum_{i,j} \Big[(i\alpha)\Psi_i^\dagger \Gamma \Psi_j + \Psi_i^\dagger \Gamma (i\alpha)\Psi_j\Big] = i\alpha\Sigma - i\alpha\Sigma = 0. \tag{F3}$$

Thus the coherence scalar is invariant.

2. F.2 Gauge transformation of the bilinear vector

Consider the emergent gauge field:

$$A_\mu(x) = \Psi^\dagger \Gamma_\mu \Psi .$$

Under $\Psi \to e^{i\alpha(x)}\Psi$ we get:

$$A'_\mu = \Psi^\dagger e^{-i\alpha}\Gamma_\mu e^{i\alpha}\Psi + (\partial_\mu \alpha)\, \Psi^\dagger \Gamma \Psi \tag{F4}$$

$$= A_\mu + \partial_\mu \alpha, \tag{F5}$$

because Γ_μ commutes with the scalar phase.

Therefore:

$$\boxed{A_\mu \to A_\mu + \partial_\mu \alpha}$$

is an *exact consequence* of the spinorial phase freedom.

3. F.3 Gauge invariance of $F_{\mu\nu}$

The field tensor is defined by necessity (Appendix B):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Under the gauge transformation:

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

we obtain:

$$F'_{\mu\nu} = F_{\mu\nu}$$

since the symmetric term $\partial_\mu \partial_\nu \alpha$ cancels.

Thus:

$$\boxed{\delta\mathcal{F} = 0 \quad \Longrightarrow \quad \text{gauge invariance}}$$

Gauge symmetry is *not* an assumption, but a direct mathematical consequence of the spinorial phase freedom in the Ψ functional.

Appendix G: Appendix G: Complete Maxwell $\to \Psi$ Mapping, Equation by Equation

The classical Maxwell system:

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho, \\ \nabla \cdot \mathbf{B} = 0, \\ \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \\ \nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J}, \end{cases}$$

is here shown to arise as the coherent second–order expansion of the universal spinorial functional (F1).

1. G.1 Definition of the gauge potential from Ψ

We define:

$$A_\mu = \Psi_0^\dagger \Gamma_\mu \Psi_0,$$

with Ψ_0 a stationary coherent configuration:

$$\left.\frac{\delta\mathcal{F}}{\delta\Psi^\dagger}\right|_{\Psi_0} = 0.$$

2. G.2 Emergent field tensor

From Appendix B:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

This is *not imposed*: it is the only antisymmetric two–form compatible with the second variation.

We identify:

$$E^i = F^{0i}, \qquad B^i = \frac{1}{2}\epsilon^{ijk} F_{jk}.$$

3. G.3 Gauss law

Compute:

$$\partial_i F^{0i} = \partial_i(\partial^0 A^i - \partial^i A^0) = -\partial_i \partial^i A^0 + \partial^0(\partial_i A^i).$$

Gauge invariance allows the Lorentz gauge:

$$\partial_\mu A^\mu = 0.$$

Thus:

$$\partial_i F^{0i} = -\Box A^0.$$

But $A^0 = \Psi^\dagger \Gamma_0 \Psi$ obeys (from the quadratic variation):

$$\Box A^0 = \rho,$$

with:

$$\rho = \Psi_0^\dagger \frac{\delta^2 \mathcal{F}}{\delta \Psi^\dagger \delta \Psi} \Psi_0.$$

Therefore:

$$\nabla \cdot \mathbf{E} = \rho.$$

4. G.4 No magnetic monopoles

The definition of B as a curl implies:

$$\nabla \cdot \mathbf{B} = 0$$

identically, because $F_{ij} = \partial_i A_j - \partial_j A_i$ is exact.

In Ψ this is the consequence of:

$$F = dA \quad \Rightarrow \quad dF = d^2A = 0.$$

5. G.5 Faraday induction

Using:

$$E^i = F^{0i}, \qquad B^k = \frac{1}{2}\epsilon^{kij}F_{ij},$$

we compute:

$$\nabla \times \mathbf{E} = \epsilon_{kij}\partial_i F^{0j} = \partial_i\partial_0 A_j - \partial_i\partial_j A_0.$$

Antisymmetry of $F_{\mu\nu}$ yields:

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}.$$

This is encoded in the Bianchi identity:

$$\partial_{[\mu}F_{\nu\rho]} = 0.$$

6. G.6 Ampère–Maxwell law

From the second variation:

$$\partial_\mu F^{\mu i} = J^i,$$

with the emergent current:

$$J^i = \Psi_0^\dagger \frac{\delta^2\mathcal{F}}{\delta\Psi^\dagger\delta(\partial_i\Psi)}\Psi_0.$$

Expanding:

$$\partial_t E^i + (\nabla \times B)^i = J^i.$$

Thus:

$$\nabla \times \mathbf{B} = \partial_t \mathbf{E} + \mathbf{J}.$$

7. G.7 Complete map

$$\boxed{\text{Maxwell equations} \quad \Longleftrightarrow \quad \delta^2\mathcal{F}[\Psi_0, \phi] = 0}$$

Each Maxwell equation corresponds to one component of the second variation of the universal Ψ functional.

[1] E. Livolsi, *The Theory of Everything*, Zenodo (2025). DOI: 10.5281/zenodo.17340645.

Aspect	**Maxwell (1865) Mechanical Theory**	**Universal Ψ Equation**
Ontology	Elastic mechanical medium (ether) filling space; fields interpreted as mechanical stress, tension, displacement.	No medium, no mechanical analogies. Spinorial field is primary; all dynamics emerge from a single functional.
Origin of EM Fields	Derived from mechanical strain and elastic displacements of the medium (*Maxwell1865*, p. 461–463).	Derived from the second variation of the universal functional (1). Maxwell equations appear as coherent-mode constraints.
Speed of Light	c^2 = elasticity/density (p. 466).	c^2 emerges from the ratio of functional derivatives $\partial^2(\Psi^\dagger\Gamma\Psi)/[\partial(\partial_t\Psi)\partial(\partial_x\Psi)]$.
Gauge Invariance	Impossible: medium displacement cannot change by a pure gradient.	Natural: $A_\mu = \Psi^\dagger\Gamma_\mu\Psi$ admits $A_\mu \to A_\mu + \partial_\mu\alpha$.
Lorentz Invariance	Incompatible: ether defines a preferred rest frame.	Derived from $\delta\Sigma = 0$ requiring antisymmetric generators $\omega_{\mu\nu}$.
Field Tensor	Not derived; written by analogy as mechanical stresses.	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ forced by second-order variation and antisymmetry of coherence.
Unification	Electrostatics, magnetostatics, and light treated separately.	All interactions emerge from a single spinorial functional, with unified coherence structure.
Energy Density	Mechanical strain energy.	Emerges from $\mathcal{H} = \Psi^\dagger\delta\mathcal{F}/\delta\Psi^\dagger - \mathcal{F}$.
Mathematical Basis	No Lagrangian; no variational principle; no operator algebra.	Full variational principle, operator structure, and functional dynamics.

TABLE I. Full structural comparison between Maxwell's 1865 theory and the Universal Ψ Equation.

Declaration of Authorship and Conceptual Integrity

This work belongs to the foundational theoretical framework of the Universal Ψ Lagrangian. Any use or reformulation must explicitly cite its origin.

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