Referee Report on
First-order relativistic hydrodynamics with an information current

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Summary

The manuscript proposes the construction of a so-called "information current" for first-order relativistic hydrodynamics, claiming that this object restores stability, causality, and thermodynamic consistency even in the ultraviolet regime. The authors present a collection of explicit constructions for several models (diffusion, electromagnetism, viscosity, Goldstone modes, chiral fluids), and use the resulting quadratic functional to compute equal-time correlators.

While the paper is technically elaborate and well written, the core conceptual claims are significantly overstated. The construction does not resolve the fundamental limitations of first-order relativistic hydrodynamics, but instead introduces ad hoc ultraviolet regulators whose physical meaning and uniqueness are not established. As a consequence, the work does not provide a genuine theoretical advance, but rather a model-dependent regularization scheme.

Major Conceptual Issues

1. Lack of a first-principles derivation

The central object of the paper, the information current E^μ, is introduced as a quadratic functional of the perturbations and their derivatives. However, its explicit form is obtained through trial-and-error adjustments, as explicitly admitted by the authors.

There is no derivation from a microscopic theory, no variational principle, and no generating functional from which E^μ uniquely follows. As a result, the construction is not predictive: different ultraviolet completions lead to different "information currents" without a criterion to select one.

2. Ultraviolet regularization is imposed, not derived

The manuscript repeatedly emphasizes that higher-derivative terms such as $\partial\phi\partial\phi$ are "irrelevant in the infrared" and may therefore be fixed arbitrarily. This argument is logically flawed.

The ultraviolet behavior determines stability, causality, and the well-posedness of the initial value problem. Introducing regulator terms by hand does not constitute a physical theory, but merely a mathematical stabilization prescription. No physical scale, cutoff mechanism, or coarse-graining procedure is identified.

3. Frame dependence remains unresolved

Although the authors claim that physical correlators are frame-independent, this statement holds only for a restricted set of observables and only after fine-tuning free parameters (e.g.

$\lambda = \tau$ in the diffusion model).

The need to tune auxiliary parameters to recover frame invariance indicates that the formalism does not eliminate the ambiguity of first-order hydrodynamics, but hides it in regulator-dependent coefficients.

Technical Concerns

4. Information current is not unique

For the same dynamical equations, multiple inequivalent information currents are constructed (e.g. in the electromagnetic case). This non-uniqueness undermines the interpretation of E^μ as a physical entropy-related quantity.

In thermodynamics, entropy currents are tightly constrained by the second law. Here, by contrast, entire families of currents satisfy the formal conditions, suggesting that the construction lacks physical rigidity.

5. Off-shell positivity is an artificial requirement

The requirement that E^μ be timelike and future-directed for arbitrary off-shell configurations is imposed by hand. In effective field theory, off-shell behavior is not physical and need not satisfy thermodynamic bounds.

Demanding global positivity for all fluctuations effectively enforces Gaussian measures by construction, rather than deriving them from statistical mechanics.

6. Equal-time correlators add no new physics

All computed correlators reproduce known results from fluctuating hydrodynamics once frame-invariant quantities are considered. No genuinely new observable prediction is produced.

In particular, delta-function correlators and Yukawa-type screening are standard results and do not validate the proposed framework.

Chiral Hydrodynamics Section

The analysis of chiral hydrodynamics explicitly demonstrates the failure of the approach: when genuine first-order chiral terms are retained, the system becomes ill-posed and unstable. The authors resolve this by setting $\xi_T = 0$ via a frame redefinition, effectively removing the chiral dynamics from the equations of motion.

This undermines the claim that the framework can handle generic first-order theories.

Overall Assessment

The manuscript provides a technically consistent but physically ad hoc procedure for stabilizing first-order hydrodynamics at the level of quadratic fluctuations. It does not overcome the known conceptual limitations of first-order theories, nor does it provide a unique or fundamental notion of information current.

The work may be of interest as a mathematical regularization scheme, but its claims should be substantially toned down. In its present form, it does not meet the standards for a foundational advance in relativistic hydrodynamics.

Recommendation

REJECT

The manuscript suffers from fundamental conceptual and methodological flaws that cannot be addressed by revision.

The core construction relies on imposing ultraviolet positivity, timelikeness of the information current, and global entropy constraints within a theory that is explicitly truncated at first order in derivatives. As demonstrated in the main text and in the Appendices, these requirements are mathematically impossible, violate effective field theory closure, and break the IR/UV matching principle.

These issues are structural and invalidate the central claims of the paper. No rephrasing, clarification, or parameter adjustment can cure these problems without abandoning the proposed framework altogether.

For these reasons, the manuscript is not suitable for publication.

A.1 General structure of first-order quadratic functionals

Let $\phi(x)$ denote a generic real-valued perturbation field (or a finite collection of fields). Consider the most general local quadratic functional constructed from ϕ and its first derivatives, truncated at first order in gradients. Its associated density can be written schematically as

$$E^0[\phi] = \frac{1}{2}\left(\phi^2 + a\,\phi\,\partial_\mu\phi\right), \tag{1}$$

where a is a constant coefficient (or matrix in field space). No term of the form $(\partial\phi)^2$ is included, as this would be second order in the derivative expansion.

The total functional is

$$E = \int d^3x\, E^0[\phi]. \tag{2}$$

A.2 Fourier-space representation

Let $\phi(x)$ be expanded in Fourier modes,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3}\,\tilde{\phi}(k)\,e^{ik\cdot x}. \tag{3}$$

In Fourier space, the functional takes the quadratic form

$$E = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\begin{pmatrix}\tilde{\phi}(-k) & k_\mu\tilde{\phi}(-k)\end{pmatrix}\begin{pmatrix}1 & \frac{a}{2}\\ \frac{a}{2} & 0\end{pmatrix}\begin{pmatrix}\tilde{\phi}(k)\\ k_\mu\tilde{\phi}(k)\end{pmatrix}. \tag{4}$$

Crucially, the quadratic form necessarily contains a zero on the diagonal of the block associated with derivatives. This is a structural consequence of the first-order truncation.

A.3 Indefiniteness at large momenta

The eigenvalues $\lambda_\pm$ of the quadratic form matrix are

$$\lambda_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 + a^2k^2}\right). \tag{5}$$

For sufficiently large $|k|$, one eigenvalue becomes negative:

$$\lambda_- \sim -\frac{|a|\,|k|}{2} \quad \text{as} \quad |k| \to \infty. \tag{6}$$

Hence, for arbitrarily high momenta, the quadratic functional E becomes unbounded from below. This implies that the Gaussian weight

$$\exp(-E) \tag{7}$$

is non-normalizable, and the ultraviolet sector of the theory is ill-defined.

A.4 Necessity of second-order terms

The only way to restore positivity is to introduce terms quadratic in derivatives,

$$E^{0}_{\mathrm{UV}} \sim b\,(\partial_\mu \phi)(\partial^\mu \phi), \quad b > 0, \tag{8}$$

which modify the quadratic form to

$$\begin{pmatrix} 1 & \frac{a}{2} \\ \frac{a}{2} & b \end{pmatrix}, \tag{9}$$

rendering it positive definite for sufficiently large b.

Such terms are necessarily of second order in gradients and therefore lie outside the strict domain of first-order hydrodynamics.

A.5 Conclusion

We conclude that:

- A quadratic functional truncated at first order in derivatives is structurally incapable of being positive definite in the ultraviolet.
- Ultraviolet positivity cannot emerge from first-order hydrodynamics alone.
- Any claim of UV stability or entropy positivity necessarily relies on the introduction of higher-derivative regulator terms.

Therefore, ultraviolet positivity in first-order relativistic hydrodynamics cannot be derived, but only imposed by hand through second-order (or higher) additions.

Appendix B: Timelike information currents and violation of EFT closure

In this appendix we show that requiring the information current E^μ to be timelike and future-directed for arbitrary gradients is incompatible with the principles of effective field theory (EFT).

B.1 EFT closure and derivative expansion

An effective field theory is defined by a controlled expansion in derivatives (or momenta) around a given infrared scale Λ_{IR}. Truncation at a given order N in derivatives implies that all higher-order terms are parametrically suppressed and physically irrelevant within the domain of validity:

$$|\partial| \ll \Lambda_{\mathrm{UV}}. \tag{10}$$

Crucially, EFTs do not make claims about ultraviolet behavior. Any quantity defined within an EFT is only required to be meaningful order by order in the expansion.

B.2 Information current as a quadratic EFT functional

The information current E^μ introduced in the manuscript is constructed as a quadratic functional of hydrodynamic fields and their derivatives, truncated at first order in gradients:

$$E^\mu = E^\mu(\phi, \partial\phi) + \mathcal{O}(\partial^2). \tag{11}$$

Such a truncation is legitimate only in the infrared regime. Demanding that E^μ remain timelike and future-directed for arbitrarily large gradients amounts to extending the validity of the truncated EFT beyond its regime of applicability.

B.3 Timelike condition as a non-local constraint

The requirement that $E^\mu E_\mu < 0$ for all field configurations imposes a global positivity constraint on the quadratic form defining E^μ. As shown in Appendix A, this constraint cannot be satisfied without adding higher-derivative terms of the form $(\partial\phi)^2$.

However, these terms are of higher order in the derivative expansion and are not fixed by the first-order EFT. Enforcing timelikeness therefore introduces non-local information about ultraviolet physics into an infrared theory.

B.4 Breakdown of EFT closure

In a closed EFT, observables at a given order depend only on coefficients defined at that order. In the present construction, instead:

- Ultraviolet stability requires adding higher-derivative regulator terms.
- The coefficients of these terms are not determined by the first-order EFT.
- Physical predictions (e.g. fluctuation spectra) depend explicitly on these regulator coefficients.

This implies that the theory is not closed under truncation: predictions at first order depend on arbitrarily chosen higher-order data.

B.5 Conceptual inconsistency

From the EFT perspective, one must choose between two mutually exclusive options:

1. Accept first-order hydrodynamics as a genuine EFT, valid only in the infrared, without imposing ultraviolet positivity.
2. Introduce a fully specified ultraviolet completion that fixes all higher-derivative terms.

The approach adopted in the manuscript does neither. It enforces ultraviolet constraints while remaining agnostic about ultraviolet physics, thereby violating EFT closure.

B.6 Conclusion

We conclude that:

- The requirement of a globally timelike information current is not an EFT-consistent condition.
- Ultraviolet positivity cannot be imposed within a first-order EFT without specifying its ultraviolet completion.

- The proposed construction therefore mixes infrared EFT logic with ultraviolet constraints, leading to a conceptual inconsistency.

As a result, the "timelike information current" should be interpreted as a model-dependent regulator choice, not as a physical consequence of first-order relativistic hydrodynamics.

Appendix C: Entropy current positivity and violation of IR/UV matching

In this appendix we demonstrate that imposing global positivity of the entropy (or information) current is incompatible with the infrared–ultraviolet (IR/UV) matching principle of effective field theory.

C.1 IR/UV matching in effective field theory

In effective field theory, infrared observables are required to match the predictions of the underlying microscopic theory order by order in the derivative expansion. Ultraviolet physics enters only through the numerical values of low-energy coefficients, not through additional constraints imposed outside the regime of validity.

In particular, IR consistency conditions (such as conservation laws and the local second law of thermodynamics) are enforced only up to the truncation order:

$$\partial_\mu S^\mu \geq 0 \quad \text{up to } \mathcal{O}(\partial^N). \tag{12}$$

No statement is made about the sign of $\partial_\mu S^\mu$ beyond this order.

C.2 Entropy current as an IR object

The entropy current S^μ (or equivalently the information current E^μ) is defined within hydrodynamics as an infrared quantity. Its constitutive relation is fixed only up to the truncation order of the gradient expansion.

Any modification of S^μ at higher order,

$$S^\mu \to S^\mu + \Delta S^\mu, \quad \Delta S^\mu = \mathcal{O}(\partial^{N+1}), \tag{13}$$

is physically invisible in the infrared and therefore unconstrained by thermodynamics.

C.3 Global positivity as an ultraviolet constraint

Requiring the entropy current to be positive (or timelike future-directed) for arbitrary field configurations and arbitrarily large gradients imposes a global constraint on S^μ that extends beyond the IR domain.

Such a requirement effectively enforces:

$$\partial_\mu S^\mu \geq 0 \quad \text{for all momenta.} \tag{14}$$

This is an ultraviolet condition, since it constrains configurations with $|\partial| \sim \Lambda_{\mathrm{UV}}$ or larger, which lie outside the hydrodynamic regime.

C.4 Failure of IR/UV matching

In a consistent EFT, ultraviolet behavior is matched by integrating out microscopic degrees of freedom, resulting in higher-order operators with coefficients determined by the UV theory.

By contrast, enforcing global entropy positivity within a first-order theory:

- introduces constraints on UV fluctuations without specifying the UV theory,
- fixes combinations of higher-order operators by hand,
- alters the ultraviolet behavior without affecting infrared matching.

This breaks IR/UV matching: the infrared theory is no longer the correct low-energy limit of any ultraviolet completion unless the imposed constraints are reproduced dynamically by the UV theory.

C.5 Incompatibility with statistical mechanics

From microscopic statistical mechanics, entropy maximization holds only for equilibrium states and small fluctuations around equilibrium. Large-gradient configurations are exponentially suppressed by microscopic dynamics, not by hydrodynamic entropy currents.

Imposing entropy positivity for all configurations replaces microscopic Boltzmann suppression with a hydrodynamic Gaussian weight, thereby encoding incorrect UV statistics.

C.6 Logical inconsistency

The framework under consideration simultaneously assumes:

1. hydrodynamics as an infrared effective description,
2. entropy current positivity as a global, all-scale constraint.

These two assumptions are mutually incompatible. Global entropy positivity cannot be derived from infrared hydrodynamics and necessarily presupposes knowledge of the ultraviolet completion.

C.7 Conclusion

We conclude that:

- Entropy (or information) current positivity is an infrared statement, valid only within the EFT domain.
- Imposing positivity beyond this domain violates IR/UV matching.
- The resulting theory cannot correspond to the low-energy limit of a consistent microscopic model.

Therefore, global entropy current positivity must be interpreted as an ad hoc regulator condition, not as a physical consequence of first-order relativistic hydrodynamics.