# Referee Report on First-order relativistic hydrodynamics with an information current

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## Referee Report

### Manuscript

Boundary Liouville Field Theory

### General assessment

The manuscript presents a detailed construction of boundary Liouville field theory based on conformal bootstrap, analytic continuation, and duality arguments. The results are technically sophisticated and of potential interest.

However, several steps that are essential for establishing the mathematical completeness of the construction are currently implicit. Clarifying these points would be important in order to assess whether the framework defines a fully consistent quantum field theory, rather than a formally constrained set of correlation functions.

Below I list a number of technical questions whose resolution appears necessary.

### 1. Existence of a generating functional

Correlation functions are constructed algebraically, without reference to an underlying functional integral. It would be important to clarify whether there exists an action functional of the form

$$S[\phi] = S_{\text{bulk}}[\phi] + S_{\text{boundary}}[\phi|_{\partial M}]$$

such that all bulk and boundary correlators can be obtained as

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \mathcal{O}_1 \cdots \mathcal{O}_n \, e^{-S[\phi]}.$$

If such a functional exists, it would be useful for the authors to provide it explicitly or to specify the precise sense in which the bootstrap construction is equivalent to a functional formulation.

### 2. Quadratic fluctuation operator

Assuming the existence of a background configuration $\phi_0$, consistency of the quantum theory requires the quadratic kernel

$$\mathcal{K}(x,y) = \left. \frac{\delta^2 S}{\delta\phi(x)\delta\phi(y)} \right|_{\phi=\phi_0}$$

to be well defined.

Could the authors clarify:

- the explicit form of the bulk–boundary quadratic operator,
- its spectrum,
- and whether it is positive semi-definite?

This information is necessary to establish stability and normalizability of the functional measure.

## 3. Boundary mode control

Boundary operators are introduced as

$$B_\beta = e^{\beta\phi}|_{\partial M}$$

with scaling dimension $\Delta_\beta = \beta(Q - \beta)$.

It would be useful to clarify how ultraviolet boundary fluctuations are controlled in the absence of an explicit boundary kinetic term derived from the action. In particular, how is the high-momentum behavior of boundary modes regulated within the proposed framework?

## 4. Analytic continuation and domain of validity

Several central quantities are defined by analytic continuation outside their original domain of convergence. While this procedure is formally consistent, it would be important to understand its physical status.

Can the authors clarify whether the analytically continued correlators correspond to a local effective description, and how this continuation is compatible with standard infrared–ultraviolet matching?

## 5. Effective field theory interpretation

If the construction is intended as an effective field theory, one would expect a derivative expansion of the form

$$G(k) = G_0(k) + \mathcal{O}\left(\frac{k}{\Lambda}\right)^n .$$

Could the authors clarify whether such an expansion exists, and how it is reconciled with the exact functional identities imposed by the bootstrap relations?

## Concluding remark

Addressing the points above would significantly clarify the mathematical status of the theory and its range of validity. In particular, an explicit discussion of the generating functional, quadratic kernel, and ultraviolet behavior appears essential for establishing the framework as a complete quantum field theory.

## Appendix A: Suggested consistency benchmark

The following consistency benchmark is proposed as a possible tool to clarify the mathematical status of the construction presented in the manuscript. It is formulated in a model-independent way and may help to assess whether the framework admits a closed functional interpretation.

### A.1 Variational formulation

As a minimal consistency check, it would be useful to verify whether the theory admits a generating functional $S[\phi]$ such that all bulk and boundary dynamics follow from the variational principle

$$\frac{\delta S}{\delta \phi(x)} = 0.$$

In particular, one may ask whether both bulk interactions and boundary terms can be obtained without introducing external potentials, fixed background structures, or independently postulated boundary parameters.

### A.2 Emergence of operator structure

A related question concerns the origin of the operator content. One may consider whether all operators entering correlation functions can be expressed as functionals of the fundamental field and its derivatives,

$$\mathcal{O}(x) = \mathcal{O}[\phi(x), \partial\phi(x)],$$

rather than being fixed by symmetry arguments or bootstrap assumptions alone.

### A.3 Quadratic fluctuation kernel

Assuming the existence of a background configuration $\phi_0$, a natural consistency requirement is the existence of a well-defined quadratic kernel

$$\mathcal{K}(x,y) = \left.\frac{\delta^2 S}{\delta\phi(x)\delta\phi(y)}\right|_{\phi=\phi_0}.$$

Clarifying the spectral properties of this operator, in particular its positivity and domain of definition in the presence of boundaries, would establish stability and normalizability of the functional measure.

### A.4 Ultraviolet behavior and locality

Another useful benchmark concerns the ultraviolet behavior of correlation functions. If the framework admits an effective description, one would expect that high-momentum behavior is governed by higher-order operators suppressed by a characteristic scale,

$$G(k) = G_0(k) + \mathcal{O}\left(\frac{k}{\Lambda}\right)^n.$$

It would be helpful to clarify how this expectation is reconciled with the exact functional relations imposed by the bootstrap construction.

### A.5 Infrared–ultraviolet matching

Finally, one may consider whether infrared consistency conditions alone are sufficient to fix ultraviolet properties, or whether additional microscopic input is required. A discussion of how IR constraints are matched to UV data would further clarify the scope and limitations of the approach.

### A.6 Purpose of the benchmark

The purpose of the above points is not to impose additional assumptions, but to provide a transparent way of assessing whether the construction defines a closed quantum field theory or should be interpreted as a formally consistent set of correlators with restricted domain of validity.